The dependence of the quantity $\mathrm{Sh}\,(\beta+1)^{1/2}P^{-1/2}$ on $|\omega_f|$ is shown in Fig. 3. This quantity remains constants for $|\omega_f| \leqslant 1$ and increases as $|\omega_f|$ increases for $|\omega_f| > 1$.

In conclusion we note that in the limiting cases of homogeneous translational ($\omega_f \to 0$) and homogeneous shear ($|\omega_f| \to \infty$) flows the expressions (4.10) and (4.11) agree with those obtained earlier in [1, 3, 4].

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STATIC AND DYNAMIC CONTACT PROBLEMS WITH COHESION

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V. A. BABESHKO
(Rostov-on-Don)

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Systems of integral equations, originating in plane and axisymmetric contact problems of elasticity theory in the case of cohesion of a stamp to a body, are studied. A method is developed which is based on factorization of matrix functions of a special kind and its foundation is given. Applications of the method in static and dynamic problems are presented. The method is especially effective in dynamic contact problems of stamp vibration on the surface of a layered medium or a cylinder.

Other methods of solving contact problems with cohesion have been proposed in [1 – 8].

1. Systems of integral equations of the following two kinds

$$\sum_{n=1}^{2} r_{mn}q_n = f_m(x), \qquad x \in \Omega, \ m = 1, 2 \tag{1.1}$$

$$r_{mn}q_n = \frac{1}{2\pi}\int_{-a}^{a}\int_{\sigma} R_{mn}(u)\,e^{iu(x-\xi)}du\,q_n(\xi)\,d\xi, \quad \Omega \equiv [-a, a] \tag{1.2}$$

$$r_{mn}q_n = \int_0^a \int_{\sigma_1} R_{mn}(u)\, J_{1-[m/2]}(ux) J_{1-[n/2]}(u\xi)\, u\xi q_n(\xi)\, du\, d\xi, \quad \Omega \equiv [0, a] \qquad (1.3)$$

are examined below.

The contour σ in (1.2), where the functions $R_{mn}(u)$ have singularities, is located in conformity with the rules established in [9, 10]. In particular, it coincides with the real axis in static problems. The contour σ_1 is the part of the contour σ lying in the right half-plane.

Let us consider the elements of the matrix $\mathbf{R}(u)$ to be regular in the domain containing the contour σ, and to possess asymptotic behavior at infinity

$$R_{mm}(u) = C\,|\,u\,|^{-1}[1 + O(u^{-1})]$$
$$R_{mn}(u) = iBu^{-1}[1 + O(u^{-1})]$$
$$C > |B|, \quad \operatorname{Im} B = 0, \quad u \to \pm\infty$$

Let us distinguish two cases: (a) the contour σ coincides with the real axis; then we consider the matrix $\mathbf{R}(u)$ to be positive-definite; (b) the contour σ has the position indicated in [9, 10]; in this case we consider the elements of the matrix $\mathbf{R}(u)$ to be connected with the real functions $K_{mn}(u)$ on the real axis by means of the relationships

$$R_{mn}(u) = K_{mn}(u), \quad R_{12}(u) = -R_{21}(u) = iK_{12}(u)$$

where the functions $K_{mm}(u)$ are even, $K_{12}(u)$ is odd and all have the same single poles $\pm\zeta_k$ $(k = 1, 2, \ldots, p)$ on the real axis.

The correct solution procedure of the systems (1.1) – (1.3) on the subspace $\overset{\circ}{H}_{-1/2}$ of some weight space [11] consisting of functions with a carrier in $[-a, a]$ or $[0, a]$ is proved easily under conditions (a). We should note that $L_\alpha \subset \overset{\circ}{H}_{-1/2}$ $(\alpha > 1)$.

The equivalence of the systems to equations of the second kind with a completely continuous operator in this subspace is proved in case (b). The author succeeded in proving uniqueness in case (b) only in L_α and under the following assumptions

1) $[K_{11}^{-1}(\zeta_r)]' > 0$
2) $[K_{11}^{-1}(\zeta_r)]'[K_{22}^{-1}(\zeta_r)]' > \{[K_{12}^{-1}(\zeta_r)]'\}^2$
3) In the case of the system (1.1), (1.2) rational functions $\Pi_{nm}(u)$, bounded at infinity and with poles at the points $\pm\zeta_k$ $(k = 1, 2, \ldots p)$ exist such that the following inequalities hold on the real axis

$$K_{11}(u)P_{11}(u) + iK_{12}(u)\,P_{12}(u) \geqslant 0$$
$$[K_{11}(u)\,K_{22}(u) - K_{12}^2(u)][P_{11}(u)\,P_{22}(u) - P_{12}(u)P_{21}(u)] \geqslant 0$$
$$P_{mn}(u) = (-1)^{m+n}\Pi_{nm}(u)[\Pi_{11}(u)\Pi_{22}(u) - \Pi_{12}(u)\,\Pi_{21}(u)]^{-1}$$

In the case of the system (1.1), (1.3) the requirement of evenness and boundedness of the functions $\Pi_{mm}(u)$ and $u\,\Pi_{mn}(u)$, $m \neq n$, is added.

Let us henceforth consider the formulated uniqueness conditions for the solutions in L_α, $\alpha > 1$ to hold.

2. Important to the construction of solutions of (1.1) – (1.3) is factorization of the matrix function $\mathbf{R}(u)$ i.e. its representation as [12]

$$\mathbf{R}(u) = \mathbf{M}_-^{-1}(u)\,\mathbf{M}_+(u) \equiv \mathbf{N}_+^{-1}(u)\,\mathbf{N}_-(u) \tag{2.1}$$

Here the matrices $\mathbf{M}_+(u)$, $\mathbf{N}_+(u)$ have elements which are regular above the contour σ, while the determinants are different from zero in this domain. Elements of the matrices $\mathbf{M}_-(u)$, $\mathbf{N}_-(u)$ possess these same properties in the domain below the contour σ. The factorization (2.1) of the matrix $\mathbf{R}(u)$ can be realized in conformity with general theorems in [12].

An approximate factorization of the matrix functions will play an important role in the construction of effective approximate solutions of the integral equations system.

In conformity with the methods in [13], let us construct the matrix $\mathbf{R}^\circ(u)$ which approximates $\mathbf{R}(u)$ in such a way that each element on the real axis satisfies the conditions of the theorem in [13]. It is easy to show that these conditions indeed assure nearness of the solutions of the systems of equations in a metric uniform with weight. Let us introduce the functional-commutative matrix $\mathbf{\Phi}(u)$ with elements $\varphi_{mn}(u)$ of the form

$$\varphi_{11} = \varphi_{22} = R_{11}(u), \qquad \varphi_{12} = -\varphi_{21} = R_{12}(u) = iK_{12}(u)$$

It is known that factorization of these matrices is carried out by means of scalar formulas by using functions of the matrices and the following representations consequently hold

$$\mathbf{\Phi}(u) = \mathbf{\Phi}_+(u)\,\mathbf{\Phi}_-(u) \equiv \mathbf{\Phi}_-(u)\,\mathbf{\Phi}_+(u) \tag{2.2}$$

The form of the elements $\varphi_{mn}^{\pm}(u)$ is given by the relationships

$$\begin{aligned} 2\varphi_{mn}^{+}(u) &= S_\pm(u) + T_\pm(u) \\ 2\varphi_{12}^{\pm}(u) &= -2\varphi_{21}^{\pm}(u) = i\,[S_\pm(u) - T_\pm(u)] \end{aligned} \tag{2.3}$$

Here $S_\pm$, $T_\pm$ are obtained as a result of factorization of the functions

$$S_+S_- = R_{11} - iR_{12}, \qquad T_+T_- = R_{11} + iR_{12} \tag{2.4}$$

relative to the contour σ.

It is easy to study the asymptotic properties of elements of the matrices $\mathbf{\Phi}_\pm(u)$. It turns out that the relationships

$$\begin{aligned} \sigma^{-1}\varphi_{11}^{\pm} &= (\mp iu)^{-\theta_\pm} + (\mp iu)^{-\theta_\mp} + O(u^{-1}) \\ \sigma^{-1}\varphi_{12}^{\pm} &= (\mp iu)^{-\theta_\pm} - (\mp iu)^{-\theta_\mp} + O(u^{-1}) \\ \sigma &= \sqrt[4]{C^2 - B^2}, \qquad \theta_\pm = 1/2 \pm i\pi^{-1}\,\mathrm{arcth}\, B/C \end{aligned} \tag{2.5}$$

are valid for them in their domains of regularity.

Now, let us introduce the matrix

$$\mathbf{H}(u) = \mathbf{\Phi}_-^{-1}(u)\,\mathbf{R}(u)\,\mathbf{\Phi}_+^{-1}(u) \tag{2.6}$$

whose elements are

$$\begin{aligned} H_{11}(u) &= 1 - 1/4\;\varphi_{12}^{-}\,[L(u)\varphi_{12}^{+} + P(u)\varphi_{11}^{+}] \\ H_{22}(u) &= 1 + 1/4\;\varphi_{11}^{-}\,[L(u)\varphi_{11}^{+} - P(u)\varphi_{12}^{+}] \\ H_{12}(u) &= 1/4\;\varphi_{12}^{-}\,[P(u)\varphi_{12}^{+} - L(u)\varphi_{11}^{+}] \\ H_{21}(u) &= 1/4\;\varphi_{11}^{-}\,[L(u)\varphi_{12}^{+} + P(u)\varphi_{11}^{+}] \\ L(u) &= (R_{22} - R_{11})[\det\mathbf{\Phi}(u)]^{-1} \\ P(u) &= (R_{21} + R_{12})[\det\mathbf{\Phi}(u)]^{-1} \end{aligned} \tag{2.7}$$

As $|u| \to \infty$, their asymptotic behavior is given by the relationships

$$H_{mm}(u) = 1 + O(u^{-1}), \quad H_{mn}(u) = O(u^{-1}), \quad m \neq n \qquad (2.8)$$

The analyticity of elements of the matrix $\mathbf{H}(u)$ as well as the estimates (2.8) show that it belongs to a dissociating algebra, and hence admits of the factorization [12]

$$\mathbf{H}(u) = \mathbf{H}_-(u)\mathbf{H}_+(u) = \mathbf{T}_+(u)\mathbf{T}_-(u) \qquad (2.9)$$

Taking (2.8) into account we obtain the first (right-sided) factorization (2.1) of the matrix $\mathbf{R}(u)$ on the basis of (2.9). The second (left-sided) factorization (2.1) of the matrix $\mathbf{R}(u)$ is constructed perfectly analogously, where it is sufficient to interchange $\mathbf{\Phi}_+$ and $\mathbf{\Phi}_-$ in (2.6) and to use the second formula in (2.9).

Let us perform the first factorization (2.9) approximately. To this end, let us represent the matrix $\mathbf{H}(u)$ by using the combination of two matrices $\mathbf{H}_k(u)$ $(k = 1, 2)$ with the even elements

$$\mathbf{H}(u) = \mathbf{H}_1(u) + u\mathbf{H}_2(u)$$

The elements of the matrices $\mathbf{H}_1$, $\mathbf{H}_2$ are bounded at infinity and can have a finite number of poles and zeros on the real axis. Let $h(u)$ be an element of one of the matrices $\mathbf{H}_k$ which is bounded at infinity, and has the poles $\pm\gamma_k$ $(k = 1, 2, \ldots, \alpha)$ and zeros $\pm\delta_k$ $(k = 1, 2, \ldots, \beta)$ on the real axis. Let us introduce a function $h_0(u)$ bounded at infinity, of the form

$$h_0(u) = h(u) \prod_{k=1}^{\gamma} \frac{u^2 - \gamma_k^2}{u^2 - \delta_k^2}$$

Here $\gamma = \max(\alpha, \beta)$. If $\alpha > \beta$ then δ_k $(k > \beta)$ are arbitrary pure imaginaries; this also occurs for $\beta > \alpha$ and γ_k $(k > \alpha)$. Therefore, $h_0(u)$ is an even continuous function on the whole axis. Let us approximate the function $h_0(u)$ on the whole axis by rational function in a uniform metric. To this end, let us map the half-axis on the segment [0, 1] by setting

$$h_0(u) = h_0\left(\sqrt{\frac{A^2 x}{1-x}}\right) \equiv g(x), \quad 0 \leqslant x \leqslant 1, \ A > 0$$

The function $g(x)$ continuous in [0, 1] is approximated to any degree of accuracy by Bernshtein polynomials, i.e.

$$g(x) \approx \sum_{s=0}^{N} g\left(\frac{s}{N}\right) C_N^s x^s (1-x)^{N-s} = g_0(x)$$

Therefore, the rational function approximating $h_0(u)$ on the whole axis is given by the relationship

$$h_0(u) \approx g_0[u^2(u^2 + A^2)^{-1}]$$

Using this method, the matrix $\mathbf{G}(u)$ with elements from rational functions approximating the matrix $\mathbf{H}(u)$ on the whole axis, is constructed.

The matrix $\mathbf{G}(u)$ with the described properties is factorized, as is known, in finite form [14]. Finally, the first approximate factorization of $\mathbf{R}(u)$ is given by the relationship (2.1) in which

$$\mathbf{M}_-^{-1}(u) \approx \mathbf{\Phi}_-(u)\mathbf{G}_-(u), \quad \mathbf{M}_+(u) \approx \mathbf{G}_+(u)\mathbf{\Phi}_+(u)$$

The second approximation (2.1) is constructed analogously.

3. By constructing the factorization of the matrix $\mathbf{R}(u)$ we reduce the system of integral equations to equations of the second kind and we construct their approximate solutions.

To this end, let us introduce the vectors

$$\mathbf{Q}(u) = \{Q_1(u),\ Q_2(u)\} \tag{3.1}$$

for the cases of (1.2) and (1.3), respectively,

$$Q_k(u) = \int_{-a}^{a} q_k(x)\, e^{iux} dx \tag{3.2}$$

$$Q_k(u) = \int_{0}^{a} q_k(x)\, x J_{1-[k/2]}(ux)\, dx \tag{3.3}$$

Moreover, let us use the notation

$$\theta_n(\alpha, \beta, a) = \frac{\alpha\beta J_n(\alpha a)\, H_{n+1}^{(2)}(\beta a) - a\alpha J_{n+1}(\alpha a)\, H_n^{(2)}(\beta a)}{\alpha^2 - \beta^2}$$

$\varkappa_k(\alpha, a, n)$ are regular functions in the lower half-plane, subject to the condition in this domain

$$i\sqrt{a}\varkappa_1 H_n^{(2)}(\alpha a) \to 1, \quad \sqrt{a}\varkappa_2 J_n(\alpha a) \to 1 \tag{3.4}$$

as $|\alpha| \to \infty$

Let $\theta(\alpha, \beta, a)$ and $\varkappa_k(\alpha, a)$ denote diagonal, second-order matrices with the elements $\theta_1(\alpha, \beta, a)$, $\theta_0(\alpha, \beta, a)$ and $\varkappa_k(\alpha, a, 1)$, $\varkappa_k(\alpha, a, 0)$, respectively.

Let us continue the system (1.1), (1.2) on the whole axis by setting the right side equal to the vector $\varphi_k(x) = \{\varphi_{1k}(x), \varphi_{2k}(x)\}$, where $k = 1$ for $x > a$, and $k = 2$ for $x < -a$.

Furthermore, let us use the notation

$$\mathbf{X}(u) = -\mathbf{M}_-(u)\, e^{-iu2a} \int_{-\infty}^{-a} \varphi_1(x)\, e^{-iu(x+a)} dx \tag{3.5}$$

$$\mathbf{Y}(-u) = -\mathbf{N}_+(u)\, e^{iu2a} \int_{a}^{\infty} \varphi_2(x)\, e^{iu(x-a)}\, dx$$

$$\mathbf{F}(u) = \int_{-a}^{a} \mathbf{f}(x)\, e^{iux} dx, \qquad \mathbf{f} = \{f_1, f_2\}$$

Let us apply a Fourier transform (generalized if the $\varphi_k(x)$ do not decrease) to the continued system (1.1), (1.2). We then arrive at the relationship

$$\mathbf{K}(u)\,\mathbf{Q}(u) = \mathbf{F}(u) - \mathbf{M}_-^{-1}(u)\, e^{-2aiu}\mathbf{X}(u) - \mathbf{N}_+^{-1}(u)\, e^{2aiu}\mathbf{Y}(-u) \tag{3.6}$$

which is valid on the contour σ.

Now repeating the known method elucidated in the one-dimensional case in [15], say, we arrive at the following system of equations of the second kind to determine $\mathbf{X}$, $\mathbf{Y}$:

$$\mathbf{X}(u) = -\frac{1}{2\pi i}\int_{\sigma} \mathbf{M}_-(-\alpha)\,[\mathbf{N}_+^{-1}(-\alpha)\, e^{-2ai\alpha}\mathbf{Y}(\alpha) + e^{-ai\alpha}\mathbf{F}(-\alpha)]\,\frac{d\alpha}{\alpha + u} \tag{3.7}$$

$$\mathbf{Y}(u) = -\frac{1}{2\pi i}\int_{\sigma} \mathbf{N}_+(\alpha)\,[\mathbf{M}_-^{-1}(\alpha)\, e^{-2ai\alpha}\mathbf{X}(\alpha) + e^{-ai\alpha}\mathbf{F}(\alpha)]\,\frac{d\alpha}{\alpha + u}$$

If $\mathbf{f}(x) \in c_2(-a, a)$, then as is easy to establish, $\mathbf{X}$, $\mathbf{Y}$ are regular in the domain below the contour σ including the contour itself, and decrease there with the weight u^γ, $\gamma < 1$.

Determining the vector functions $\mathbf{X}$, $\mathbf{Y}$ from (3.7), we then find $\mathbf{Q}$ from (3.6), and besides, $q_k(x)$ as well by taking account of (3.2). By virtue of the equivalence of the system (1.1), (1.2), uniqueness holds for the system (3.7) in the mentioned class. Since the integral operators are completely continuous in the space of functions which are continuous on the contour σ with weight, as is easily confirmed, then the system (3.7) is solvable.

Repeating the same reasoning in the case of the system (1.1), (1.3), the case (b), and applying Bessel transforms of the first and zero orders, respectively, to the first and second equations, we arrive at a vector relationship on the contour σ such as

$$\mathbf{K}(u)\,\mathbf{Q}(u) = \mathbf{F}(u) + \frac{1}{2\pi i}\int_\sigma \mathbf{\Theta}(u, \beta, a)\,\mathbf{N}_+^{-1}(\beta)\,\varkappa_1(\beta, \alpha)\,\mathbf{Z}(\beta)\,d\beta \tag{3.8}$$

$$\mathbf{F}(u) = \{F_1, F_2\}, \quad F_k = \int_0^a f_k(r)\,rJ_{1-[k/2]}(ur)\,dr$$

Applying the same methods to (3.8) as in the reduction of the relationship (3.6) to the system (3.7) by using (3.4), we obtain the following equation of the second kind to determine $\mathbf{Z}$:

$$\mathbf{Z}(u) = \frac{1}{4\pi^2}\int_{\Gamma_1}\int_{\Gamma_2} \mathbf{N}_+(\alpha)\,[\mathbf{C}(\alpha, \beta)\,\mathbf{N}_+^{-1}(\beta)\,\mathbf{Z}(\beta) + \varkappa_2(\alpha, a)\,\mathbf{F}(\alpha)\,2\alpha] \times \frac{d\beta\, d\alpha}{(\alpha - u)(\alpha^2 - \beta^2)} \tag{3.9}$$

$$\mathbf{C}(\alpha, \beta) = (\alpha + \beta)\,\mathbf{I} - \mathbf{\Theta}(\alpha, \beta, a)\,\varkappa_1(\beta, a)\,\varkappa_2(\alpha, a)\,(\alpha^2 - \beta^2) \tag{3.10}$$

$$u > \Gamma_1 > \Gamma_2$$

The contours Γ_1, Γ_2 are located in the domain of regularity of the matrix functions $\mathbf{R}(u)$. The sense of the inequalities (3.10) has been clarified in [16]. By simple manipulations (3.9) is also reduced to an equation with completely continuous operator in the same space as for $\mathbf{X}$, $\mathbf{Y}$ and its single-valued solvability for $\mathbf{f}(r) \in C_2$ is proved analogously. The form of (3.9) is convenient for the construction of an approximate solution.

4. To construct approximate solutions of the systems of integral equations (1.1), let us perform an approximate factorization of the function (2.4). To this end, let us apply approximations analogous to those used in [17]. Namely, to construct the first factorization of (2.4), let us set

$$h_k(u) = (R_{11} - iR_{12})D_+^{-1}(u, k)D_-^{-1}(u, k) = h_{k1}(u) + uh_{k2}(u) = 1 + O(u^{-1}), \quad k = 1,2 \tag{4.1}$$

$$D_+(u, 1) = \sigma(b - iu)^{-\theta_+}, \quad D_-(u, 1) = \sigma(b + iu)^{-\theta_-}, \qquad b > 0 \tag{4.2}$$

$$\begin{aligned} &D_+(u, 2) = \sigma^2\beta^{-1}\Gamma(g_2/\beta - iu/\beta)\Gamma^{-1}(b_2/\beta - iu/\beta) \\ &D_-(u, 2) = \sigma^2\beta^{-1}\Gamma(g_1/\beta - iu/\beta)\Gamma^{-1}(b_1/\beta - iu/\beta) \\ &b_1 - g_1 = {}^1/_2\,\beta\pi^{-1}(\pi + i\ln\lambda) \\ &b_2 - g_2 = {}^1/_2\,\beta\pi^{-1}(\pi - i\ln\lambda), \quad \lambda = (C + B)(C - B)^{-1} \end{aligned} \tag{4.3}$$

Here the functions $h_{ks}(u)$ are even, and can be approximated by rational functions by using the method expounded in Sect. 2. Therefore, the function $h_k(u)$ approximated by a rational function is easily factorized, and besides, the first approximate factorization of (2.4) is easily constructed by taking account of (4.2), (4.3). Hence

$$S_+ \approx D_+(u, k)h_k^+(u), \qquad S_- \approx D_-(u, k)h_k^-(u)$$
$$h_k(u) \approx h_k^+(u)h_k^-(u)$$

where $h_k^{\pm}(u)$ are rational functions bounded at infinity.

Furthermore, applying methods expounded in Sect. 2 to the factorization of the matrix functions $\mathbf{R}(u)$, we see that the elements of the matrices $\mathbf{R}_{\pm}(u)$ are combinations of the functions $D_{\pm}(u, k)$ multiplied by rational functions.

This circumstance permits application of a method associated with deformation of the contours in the lower half-plane for obtaining the approximate solutions of (3.7), (3.9). If the contour intersects poles of the integrands when deformed, then residues will appear in the right sides of the relationships (3.7), (3.9) and the integrals over the contours deformed in the lower half-plane will decrease. When sufficiently small, these latter can be neglected in constructing the approximate solutions. As a result, exactly as in the one-dimensional case [15], the construction of the approximate solutions of (3.7), (3.9) reduces to solving a finite system of linear algebraic equations. Let us note that since branch points $\pm ib$ are encountered when using the approximating functions $D_{\pm}(u, 1)$, then the number b must be selected as large as possible for deformation of the contours in the lower half-plane at a sufficient distance.

Exactly as in [18], to describe the solution in the neighborhood of the stamp edges it is expedient to use the approximation (4.2), while (4.3), correspondingly, for the inner domain.

Omitting the calculations, let us present the general form of the approximate solutions of the integral equations (1.1) – (1.3) in the inner domain and in the neighborhood of the edges.

In the case of (1.2) we have

$$q_k(x) = \sum_{r=1}^{N} [c_{rk}e^{iz_r(k)(a-x)} + d_{rk}e^{i\zeta_r(k)(a+x)}], \quad x \in (-a, a)$$
$$q_k(x) = c(a \mp x)^{-\theta_\pm}, \quad x \to \pm a$$

In the case of (1.3)

$$q_k(x) = \sum_{r=1}^{N} s_{rk} I_{1-[k/2]}[z_r(k)x], \quad x \in [0, a)$$
$$q_k(x) = s(a - x)^{-\theta_+}, \quad x \to a$$

Here $z_r(k)$ are poles of the elements of the matrix $\mathbf{N}_-^{-1}(u)$ corresponding to the upper row for $k = 1$ and the lower row for $k = 2$. The $\zeta_r(k)$ and the matrix $\mathbf{M}_+^{-1}(u)$ are similarly interrelated.

As illustrations, let us present the matrix-function $\mathbf{R}(u)$ encountered in problems about the effect of stamps on an elastic layer in the dynamic and static cases.

The problem is the vibrations of a stamp lying without friction on a rigid base

$$R_{11} = -{}^1/_2\, \varkappa_2^2\sigma_2 \operatorname{ch}\sigma_1 \operatorname{ch}\sigma_2\Delta^{-1}(u)$$

$$R_{22} = -{}^1/_2\ \varkappa_2{}^2\sigma_1 \operatorname{sh}\sigma_1 \operatorname{sh}\sigma_2\Delta^{-1}(u)$$
$$R_{12} = -R_{21} = i\,[(u^2 - {}^1/_2\ \varkappa_2{}^2)\operatorname{sh}\sigma_2 \operatorname{ch}\sigma_1 - \sigma_1\sigma_2 \operatorname{sh}\sigma_1 \operatorname{ch}\sigma_2]\Delta^{-1}(u)$$
$$\Delta(u) = (u^2 - {}^1/_2\varkappa_2{}^2)^2 \operatorname{ch}\sigma_1 \operatorname{sh}\sigma_2 - u^2\sigma_1\sigma_2 \operatorname{sh}\sigma_1 \operatorname{ch}\sigma_2$$
$$\sigma_k = \sqrt{u^2 - \varkappa_k{}^2},\ \varkappa_1{}^2 = \rho\omega^2h^2(\lambda + 2\mu)^{-1},\ \varkappa_2{}^2 = \rho\omega^2h^2\mu^{-1}$$

where ρ, λ, μ are the density of the layer material and the Lamé coefficients, respectively, h is the layer thickness, and ω is the frequency of stamp vibration .

It is seen that the matrix $\mathbf{R}$ refers to case (b). Problems about the vibrations of stamps cohering to a layered medium and elastic cylinders reduce to systems with analogous matrices.

Case (a) holds in the static version ($\omega = 0$) of the considered problem, in problems of rigid cohesion of stamps to elastic wedges, in hydrodynamic problems of the vibrations of plates on a viscous fluid surface.

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ON STATIC AND DYNAMIC COMPUTATIONS OF ONE-DIMENSIONAL REGULAR SYSTEMS

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M. L. BURYSHKIN
(Odessa)

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Simplifications of the computations of statics and the small vibrations of regular mechanical structures are investigated. On the basis of the method of an elemenatry cell it is shown that these simplifications hold for all regular systems which are representable as elementary in the sense of some irreducible representation of the subgroup $D_{2h}^{(1)} \subset D_{2h}^{(1)*}$, where $D_{2h}^{(1)*}$ is the space symmetry group of the corresponding infinite regular system. The boundary conditions of such elementary systems are described in general form. The essence of the simplifications is the passage from a computation of the regular construction over to computations of a finite number of elementary systems in the sense of the group $D_{2h}^{(1)*}$ whose types are indicated. The loading of the elementary systems is defined by using a developed effective method of decomposing the load of the initial regular system.

A number of investigations [1 – 4] is devoted to a study of regular mechanical systems. These investigations are associated with translational symmetry of an infinite regular system in [2], which permitted use of the group representation theory apparatus developed for applications [5]. However, the most general and complete results in the mechanics of regular systems should be expected in a more perfect accounting of the symmetry elements of an infinite regular system.